Presentation and Syllabus [1]

1. The information contained in this document may change from time to time as developments occur in Paris-Saclay University.

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Introduction

The Master 1 "Mathematics in interactions" (MINT) of the University of Versailles SQY targets students able to follow a solid and modern graduate curriculum in mathematics and their applications. It is part of a the M.S. "Mathematics applications" of University Paris-Saclay.

The main objective of this master is to train high-level mathematicians mastering many different skills going form algebra and analysis to modelling and programming, and that will allow them to consider a wide range of career possibilities.

The program is build around a common core and two specializations. The core syllabus focuses upon a set of fundamental knowledge in mathematics, computer science and modelling. The two proposed specializations are "Applied algebra" and "Modelling and simulation".

After the first year, students may be admitted to the second year of the master program « Mathematics and Applications » of University Paris-Saclay and two specializations are then suggested : "Analysis, Modelling and Simulation", "Analysis, Arithmetics, Geometry and Applied algebra".

This document sets out the outline of the courses of the common core and of both the specializations. Enclosed a list of the professors down the document.

Specialization M1 "Applied algebra"

Students specializing in Applied algebra will take extensive courses in commutative algebra, in arithmetics, in cryptography and algebraic theory of systems.

Specialization M1 "Modelling and simulation"

Students specializing in Modelling and simulationwill take extensive courses in the field of partial differential equations, in optimization, in scientific computing and modelling.

Both specializations open new professional doors in academic and economic fabrics. Students can apply for a Phd thesis in applied or in pure mathematics. They will also be able to apply for a job as an engineer in applied mathematics or in computer science.

Core courses

- Probabilities (3 ects)
- Computer aided mathematics (6 ects)
- Algorithm analysis and programming (5 ects)
- Information theory (4 ects)
- Project module (6 ects)
- English (3 ects)

Courses of the specialization "Applied Algebra" (first year)

- Algebra 1 (6 ects)
- Number theory and cryptography (6 ects)
- Cryptography (6 ects)
- Algebra 2 (6 ects)
- Introduction to elliptic curves (6 ects)
- Automatic control and algebra (3 ects)

Courses of the specialization "Modelling and simulation" (first year)

- Introduction to functional analysis and partial differential equations (6 ects)
- Optimization (6 ects)
- Analysis of partial differential equations (6 ects)
- Advanced numerical methods and programming (6 ects)
- Stability and bifurcation (3 ects)
- Modelling in mechanics (6 ects)

Courses

of the first year

(Master *Mathematics in Interactions*)

Core courses

27 ECTS (12 ECTS in Semester 1, 15 ECTS in Semester 2)

Probability

Institution: Departement of Mathematics, UVSQ
Teaching hours: Lectures: 12h   Tutorials: 12h
ECTS: 3
Semester: 1
Nature: Compulsory
Faculty members: Catherine Donati-Martin and Alexis Devulder
Location: UVSQ
Specializations: Modelling and simulation, Applied Algebra
Examination: midterms and final exam

Prerequisites: Integral calculus, measure theory ; Probability (L3 lecture)

Description

We review the concepts of probability space and random variables. We develop different notions of convergence of random variables. The notion of stochastic processes (family of random variables indexed by a time parameter) is introduced that we illustrate by the study of random walks and Markov chains.

Syllabus

- Probability spaces, random variables, independence
- Convergence of random variables, limit theorems : law of large numbers, central limit theorems
- Conditional expectation
- Random walks
- Markov chains

References

- P. Barbe et M. Ledoux, *Probabilité*, Belin, 1998.
- B. Bercu et D. Chafaï, *Modélisation stochastique et simulation. Cours et applications*, Dunod, 2007.
- R. Durrett, *Probability : Theory and Examples*, Duxbury, 2005.
- D. Foata et A. Fuchs : *Calcul des Probabilités : Cours, exercices et problémes corrigés*, Dunod, 2003.
- Olivier Garet, Aline Kurtzmann, *De l'intégration aux probabilités*, Ellipses, 2011.
- P. Baldi, L. Mazliak et P. Priouret *Martingales et Chaînes de Markov.* Hermann, collection Méthodes, 1998.
- W. Feller, *An introduction to probability theory and its applications*, Wiley.

Computer aided mathematics

Institution: Department of Mathematics, UVSQ
Teaching hours: Lectures: 27h   Tutorials: 24h
ECTS: 6
Semester: 1
Faculty members: Guillermo Moreno-Socías
Location: UVSQ
Nature: Compulsory
Specializations: Applied Algebra, Modelling and simulation
Examination: midterms and final exam

Prerequisites: undergraduate algebra

Description

The course is made up of two main parts.
The first part is devoted to an introduction of Computer algebra. This consists of studying methods which allow to achieve results in an exact, actual and efficient manner. The basic tool is the *algorithm*. Various kinds of algorithms will be discussed in detail in this first part of the course. Algorithmic efficiency will also be addressed through complexity estimate.
The course combines conceptual presentations with practical hands-on computer sessions using Sage, Maple, etc.

Syllabus

Symbolic Calculus

- Basic objects : big integers, univariate polynomials.
- Representation. Addition and substraction. Multiplication. Euclidean division.
- Euclide's algoirthm : GCD, Bézout's identity. Applications.
- Modular arithmetic. Chinese remainder theorem.
- Evaluation and interpolation (Legendre's polynomials). Change of Representation
- Fast Multiplication : Karatsuba algorithm ; Fast Fourier transform.
- Fast euclidean division with Newton's method.
- Fats evaluation and interpolation. Fast chinese remainder theorem.
- Fast Euclide's algorithm.
- Fast linear algebra : matrix multiplication with Strassen algorithm.

- Polynomial factorization over finite fields (Gauss).

Numerical linear algebra

- Reminders on direct methods (*LU* method, Cholesky, QR,..) and on classical iterative methods (Jacobi, Gauss-Seidel, relaxation).
- Modern iterative methods. Méthodes itérarives modernes. Krylov Subspace Methods
  - Conjugate gradient method.
  - GMRES method.
  - Biconjugate gradient stabilized method.
  - Preconditioning.
- Computation of eigenvalues : Lanczos's algorithm, Arnoldi and Jacobi-Davidson methods.

References

- J. Von zur Gathen & J. Gerhard, *Modern Computer Algebra* , *3rd Edition* , *Cambridge University Press* (2013).
- V. Shoup, *A Computational Introduction to Number Theory and Algebra* , *2nd Edition* , *Cambridge University Press* (2008).
- J. Stoer et R. Bulirsh, Introduction to numerical analysis, Springer (2nd edition).

Algorithm analysis, Programming

Institution: Department of Computer Science, UVSQ
Teaching hours: Lectures: 20h Tutorials: 20h
ECTS: 5
Semester: 2
Faculty members: Luca De Feo
Location: UVSQ
Language: French or english
Nature: Compulsory
Specializations: Modelling and simulation, Applied Algebra, Computer Science
Examination: midterms and final exam

Objectives : basic knowledge on the design of efficient algorithms for scientific and symbolic computing.

Description

Introduction to algorithm design and performance analysis. Computer lab tutorials use the Python/Sage environment.

Added values : Python programming.

Syllabus

- Algorithm analysis, complexity models, asymptotic complexity, complexity classes.
- Data structures and algorithms : sorting, stacks, queues, hash tables, trees, graphs.
- Dynamic programming, integer linear programming.
- Arithmetic algorithms : multiplication, gcd, matrix multiplication.
- Geometric algorithmes : linear programming, Vornoï diagrams.

References

- Thomas H. Cormen. Charles E. Leiserson. Ronald L. Rivest. Clifford Stein. Introduction to Algorithms. Third Edition. The MIT Press. Cambridge, Massachusetts.
- Christos H. Papadimitriou. Computational complexity. Addison-Wesley, 1994. 523 pages.

Information and coding theory

Institution: Department of Computer Science, UVSQ
Teaching hours: Lectures: 16h Tutorials: 16h
ECTS: 4
Semester: 2
Faculty members: Michaël Quisquater
Location: UVSQ
Language: French
Nature: Compulsory
Specializations: Modelling and simulation, Applied Algebra, Computer Science
Examination: midterms and final exam

Prerequisites: Algebra and Linear Algebra at undergraduate level : modular arithmetic, computation in finite fields. Basic principles of probability theory. Basic knowledge of algorithmic.

Objectives : The goal is to present an overview of the concepts in information theory as well as a descriptions of different error correcting codes. A particular focus will be devoted to the decoding algorithms.

Description

The purpose of a communication system is the transport of information from a source to a receiver via a communication channel. This channel generally has imperfections which can cause transmission errors. Also, the channel may be subject to eavesdroppers that may pose privacy issues. Finally, it is important to optimize the use of a channel because of its cost. To meet these requirements, a preprocessing of the information is carried out ; it is the chain coding . It is divided into three steps : compression , encryption and adding redundancy. These techniques use probability theory and discrete algebra. This course presents the basics of the first and third stage of the chain coding, the second being extensively studied in cryptography courses.

Syllabus

- Basics of information theory (entropy, mutual information).
- Lossless compression algorithms (step 1 in the chain coding).
- Theory of error correcting codes (step 3 in the chain coding). Discrete-time memoryless channel. Concept of capacity. Noisy-channel coding theorem. Principle of maximum likelihood decoding. Bound on the decoding error probability.
- Theory of block error-correcting codes. Minimum distance problem and bounds on the size of a code. Notion of perfect code.

– Linear codes. Generator matrix and parity matrix. Syndrome decoding. Dual codes. Weight enumerator polynomial. Mac-Williams identities.
– Study of certain classes of linear codes (block) and decoding algorithms .
– Convolutional codes and the Viterbi algorithm.

References

– The Theory of Error-Correcting Codes. F. J. MacWilliams, N. J. A. Sloane North Holland Publishing Co. 1977.
– Théorie des codes (Compression, cryptage, correction). J.-G. Dumas, J.-L. Roch, E. Tannier et S. Varrette, Dunod 2007.

Project module

Institution: Department of Mathematics, UVSQ
ECTS: 6
Semester: 1
Location: UVSQ
Language: French
Nature: Compulsory
Specializations: Modelling and simulation, Applied Algebra
Evaluation: Quality of work, writing a report, defense before a jury (20 minutes + jury's questions).

Prerequisites: prerequisites depend on the chosen subject.

Description
The project, realized in pairs or solo, is an opportunity for students to do an in-depth investigation into a subject of their own choice, in accordance with a tutor. The subject could be the understanding of an outstanding mathematical result or the study of a volume. Projects involving industry cooperation could also be considered.

The tutor will guide the students during the different stages of this project. A report must be drawn up and defended during twenty minutes before a jury. The report and the presentation are marked.

English

Institution: Institut d'Etudes Culturelles et Internationales
Teaching hours: Lectures: 0h Tutorials: 27h
ECTS: 3
Semester: 1
Faculty members: Florian Leniaud and Jean-Baptiste Goyard
Location: UVSQ
Language: English
Nature: Compulsory
Specializations: Modelling and simulation, Applied Algebra, Computer Science
Examination: midterms and final exam

Prerequisites:

- Be able to conduct oral presentations
- Be able to understand written scientific and general English
- Have a good knowledge in English grammar.

Description

Acquiring professional communication skill :

- work in team
- individual Power Point presentations
- arguing and counter-arguing.

Syllabus

- Job Interview
- Debating
- CV - Cover letter - Essay writing
- Listening Comprehension
- TOEIC training

Specialization "Applied Algebra"

33 ECTS (18 ECTS in Semester 1, 15 ECTS in Semester 2)

Algebra 1

Institution: Department of Mathematics, UVSQ
Teaching hours: Lectures: 24h Tutorials: 24h
ECTS: 6
Semester: 1
Faculty members: Martin Andler
Location: UVSQ
Language: french
Nature: Compulsory
Specializations: Applied Algebra
Examination: midterms and final exam

Prerequisites: undergraduate algebra

Description
After a brief review of the basic structures of abstract algebra (Groups, Rings, Fields), we study polynomial algebras over a field and a ring, and field extensions. The main aims of the course are Galois theory, and modiule theory, in particular the structure of modules over a principal ring.

Syllabus

- Rings of polynomials on a ring and a field ; heredity theorems
- Field extensions ; algebraic, normal, separable extensions
- Galois group of an extension ; Galois correspondence
- Modules over a ring, finitely genrated modules ; modules over a principal ring.

References

- Lang S., *Algebra*, Springer.
- Stewart I., *Galois Theory*, Chapman & Hall.
- Chambert-Loir A., *Algèbre corporelle*.

Number theory and Cryptography

Institution: Department of Mathematics, UVSQ
Teaching hours: Lectures: 24h Tutorials: 24h
ECTS: 6
Semester: 1
Faculty members: Vincent Sécherre
Location: UVSQ
Language: French
Nature: Compulsory
Specializations: Applied Algebra
Examination: midterms and final exam

Prerequisites: Notions on groups, rings, fields, polynomials and congruences.

Objectives : The goal of this course is to show how algebraic properties can be used in order to prove results in Arithmetics, with applications to Cryptography. The notions studied in this course will be required for the course of Cryptography, the course "théorie de l'information et du codage" and the courses on elliptic curves of 1st and 2nd year.

Description

The goal of this course is to show how algebraic properties (in particular properties of certain groups and rings) can be used in order to prove results in Arithmetics, with applications to Cryptography. In a first part, I will recall some basic facts about groups and rings that will be useful to us. In particular, we will study integers and polynomials (in one indeterminate and with coefficients in a field), emphasizing on their common algebraic properties. We will then study congruence rings and finite fields (including Gauss's Quadratic Reciprocity Law) and we will see how to deduce some primality tests from there. We will then study various properties on rings (Euclidean, Principal, Integrally closed rings, Unique Factorization Domains, etc.) and show how such properties may have consequences in Arithmetics. In particular, we will prove the Theorem on sums of two squares by using Gauss's integers. We will then introduce the notion of quadratic integer, study quadratic units and solve Pell's equation $x^2+dy^2=1$.

Syllabus

- Groups and rings
- Integers and polynomials
- Congruences
- Finite fields
- The Gauss integers and the Two Squares Theorem

- Euclidean, Principal rings and Unique Factorization Domains
- Quadratic units and Pell's equation

References

- K. Ireland et M. Rosen, *A classical introduction to modern number theory*, Graduate texts in mathematics **84**, Springer, 1990.
- P. Samuel, *Algebraic Theory of Numbers*, translated from French by A. Silberger, Hermann, Paris, 1970.

Cryptography

Institution: Department of Computer science and department of Mathematics, UVSQ
Teaching hours: Lectures: 20h Tutorials: 20h
ECTS: 6
Semester: 1
Faculty members: Louis Goubin
Location: UVSQ
Language: French
Nature: Compulsory
Specializations: Applied Algebra, Computer Science
Examination: midterms and final exam

Prerequisites: Algebra and Linear Algebra at undergraduate level : modular arithmetic, computation in finite fields. Basic principles of probability theory and statistics. Basic knowledge of algorithmic.

Description

The goal is to provide an overview of the main algorithms used for encryption, authentication and electronic signature, and how to use them to secure digital communications.
Upon successful completion of this course, students should be able to :

- apply modular arithmetic and basic finite field operations related to cryptographic techniques
- describe basic concepts and algorithms of cryptography, including encryption/decryption, hash functions, and public-key cryptography
- evaluate the security of cryptographic primitives
- create and analyze protocols for various security objectives

Syllabus

- Secret-key cryptography, Public-key cryptography
- Brute-force attacks, replay attacks
- Known-ciphertext attacks, chosen-plaintext attacks, chosen-plaintext-and-ciphertext attacks
- Interactive and non-interactive attacks
- Stream ciphers, Block ciphers
- Transposition and substitution, Feistel schemes
- DES, AES
- One-way functions, Hash functions
- Key-exchange algorithms
- RSA, Zero-knowledge algorithms

– Applications

References

– N. Koblitz, *A Course in Number Theory and Cryptography*, GTM 114, Springer, 1994.
– A.J. Menezes, P.C. van Oorschot, S.A. Vanstone, *Handbook of Applied Cryptography*, CRC Press, 1997.
– D. Stinson, *Cryptography : Theory and Practice*, Third Edition (Discrete Mathematics and Its Applications), CRC Press, 2005.
– S. Vaudenay, *A Classical Introduction to Cryptography : Applications for Communications Security*, Springer, 2005.

Algebra 2

Institution: Department of Mathematics, UVSQ
Teaching hours: Lectures: 24h Tutorials: 24h
ECTS: 6
Semester: 2
Faculty members: Vincent Cossart
Location: UVSQ
Language: french
Nature: Compulsory
Specializations: Applied Algebra
Examination: midterms and final exam

Prerequisites: basics notions on groups, rings, polynomials

Description

The main objective is to introduce algebraic geometry and formal calculus. The main topics are the polynomial rings. We will always interpret abstract algebra in the langage of algebraic geometry : Normalization Lemma vs. projection on a vector space, Nullstellensatz vs. computation of the ideal of a closed algebraic set, geometric interpretation of Krull dimension.

Syllabus

- Noetherian rings, Hilbert Basis Theorem.
- Zariski's Topolgy of k^n.
- Correspondence between ideals and algebraic closed sets.
- Rings of fractions, localization.
- Integral extensions : going up et going down.
- Normalization Lemma, transcendance degree, dimension.
- Nullstellensatz.

References

- Atiyah et Mac Donald, *An introduction to commutative algebra*, Addison-Wesley, 1969.
- Chambert-Loir *Algèbre commutative et introduction a la géométrie algébrique* http ://www.math.u-psud.fr/ chambert/enseignement/2013-14/aceiga/Dea.pdf
- Cox, Little et O'Shea, *Ideal, varieties and algorithms*, Springer, 1991.
- Matsumura, Hideyuki *Commutative ring theory.* Cambridge University Press, 1986.
- C. Peskine *An algebraic introduction to complex projective geometry, I. Commutative algebra*, Cambridge University Press, 1996.
- D. Perrin, *Cours d'algèbre*, Ellipses, 1996.
- Samuel et Zariski, *Commutative algebra*, 2 volumes, Springer.

Introduction to elliptic curves

Institution: Department of Mathematics, UVSQ
Teaching hours: Lectures: 24h   Tutorials: 24h
ECTS: 6
Semester: 2
Faculty members: Mohamed Krir
Location: UVSQ
Language: French
Nature: Compulsory
Specializations: Applied Algebra
Examination: midterms and final exam

Prerequisites: Basics of number theory

Description
Tha aim of this course is an elementary introduction to the theory of elliptic curves with focus on explicit calculus. We first define and introduce some proprieties of the projective plane over a field K. Then, we define elliptic curves over K, the group law, rationnal functions and divisors. The last part is reserved to notion of morphism, isogenies, torsion points and Hasse theorm.

Syllabus
- Function on the projective line
- Elliptic curves
- Rationnal functions
- Divisors on elliptic curves
- Morphism of elliptic curves
- Isogenies
- Torsion points
- Weil pairing
- Hasse theorem

References

- J. H. Silvermann, *The arithmetic of elliptic curves*, Springer 1986.
- *Any other book introducing elliptic curves*

Automatic control and Algebra

Institution: Department of Mathematics, UVSQ
Teaching hours: Lectures: 16h Tutorials: 16h
ECTS: 3
Semester: 2
Faculty members: Alban Quadrat
Location: UVSQ
Language: French
Nature: Compulsory
Specializations: Applied Algebra
Examination: midterms and final exam

Prerequisites: Basics of algebra (groups, rings, fields, polynomials and congruences)

Description

Nowadays, interconnected systems play a more and more important role in our daily lives. The mathematical systems theory aims at studying general systems (physical, biological, economical, . . .), their interconnections and their control. In this course, we will give an introduction to this theory for dynamical systems defined by linear ordinary differential equations and difference equations. In particular, we will show how module theory can be used to characterize important properties of control systems (controllability, observability, flatness, . . .) and to develop control laws for these systems. To this end, we shall study algorithmic aspects of module theory for skew polynomial rings of functional operators (differential operators, shift operator, delay operator). These results can be implemented in computer algebra systems such as `Maple` ou `Mathematica`.

Syllabus

- Skew polynomial rings, Ore extensions
- Module theory over a principal ideal domain
- Introduction to module theory
- Introduction to homological algebra
- Continuous and discrete linear dynamical systems
- Stability and stabilization
- Controllability, observability, stabilisability, detectability
- Interconnections and control laws (feedback)
- Introduction to optimal control and robust control

References

- D. Eisenbud, *Commutative Algebra With a View Toward Algebraic Geometry*, Graduate Texts in Mathematics, vol. 150, Springer-Verlag.

– T. Kailath, *Linear Systems*, Prentice-Hall Information and System Sciences Series, 1979.
– J. M. Polderman, J. C. Willems, *An Introduction to Mathematical Systems Theory : A Behavioral Approach*, Texts in Applied Maths 26, Springer, 1997, http://wwwhome.math.utwente.nl/~poldermanjw/onderwijs/DISC/mathmod/book.pdf.
– J. J. Rotman, *An Introduction to Homological Algebra*, Universitext, seconde édition, Springer, 2009, http://archive.org/details/AnIntroductionToHomologicalAlgebra2ndRotman.

Specialization "Modelling and simulation"

33 ECTS (18 ECTS in Semester 1, 15 ECTS in Semester 2)

Introduction to functional analysis and partial differential equations

Institution: Department of Mathematics, UVSQ
Teaching hours: Lectures: 24h Tutorials: 24h
ECTS: 6
Semester: 1
Nature: Compulsory
Language: French
Faculty members: Pierre Gabriel
Location: UVSQ
Specializations: Modelling and simulation, Mathematical Methods for Mechanics
Examination: midterms and final exam

Prerequisites: Functions of several variables, Differential calculus, Integral calculus, Normed vector spaces

Objectives : The objective is an initiation to functional analysis and to its applications in studying partial differential equations.

Description
This course will start with an introduction to hilbertian analysis and distribution theory. These tools will be used to analyse some elliptic partial differential equations issued from physics and mechanics.

Added values : After this course, the students will have some theoretical basis required for studying partial differential equations issued from physics and mechanics.

Syllabus

- Recalls and complements about normed vector spaces.
- Hilbert spaces, orthogonal projection, hilbertian basis.
- Theorem of Riesz-Fréchet, Theorem of Lax-Milgram.
- Elements of distributions. Fourier transform.
- L^2 space. H^m Sobolev spaces.
- Traces and Green formula.
- Poincaré and Poincaré-Wirtinger inequalities.
- Examples of partial differential equations. Poisson equation. Fondamental solution.
- (Stationnary) linear elasticity equations. Korn Inequality.

References

1 Haïm Brézis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, 2010.

2 Lawrence C. Evans, Partial differential equations, Graduate Studies in Mathematics, Vol. 19, AMS.

3 Mathematical Elasticity : Three-dimensional elasticity, Volume 1. Elsevier, 1993.

Optimization

Institution: Department of Mathematics, UVSQ
Teaching hours: Lectures: 24h Tutorials: 24h
ECTS: 6
Semester: 1
Faculty members: Tahar Z. Boulmezaoud and Laurent Dumas
Location: UVSQ
Language: French
Nature: Compulsory
Specializations: Modelling and simulation, Mathematical Methods for Mechanics
Examination: midterms and final exam

Prerequisites: Multivariable calculus, basic elements of differential calculus

Description

Several problems in industry, in physics and in economics consist to minimize or to maximize an objective function. The objective of this course is to provide a number of theorical and practical for handling these issues. The first part of the course is devoted to some central elements of constrained and unconstrained optimization theory. In the second part, focus is on numerical deterministic and stochastic optimization methods.
The course could combine conceptual presentations with practical hands-on computer sessions.

Syllabus

- Introduction. Examples.
- Convexity : convex sets, convex functions convexes, properties.
- Unconstrained Optimization : first and second order optimality conditions. The case of convex programming.
- Constrained Optimization : Karush-Kuhn-Tucker theorem, Lagrange multipliers.
- Numerical methods : descent methods (gradient descent, quasi-Newton methods,..etc), Stochastic methods (simulated annealing, evolutionary algorithms,...).

References

- Ph. G. Ciarlet, Introduction à l'analyse numérique matricielle et Optimisation, Masson, 1988.
- N. Gould, S. Leyffer, An introdution to algorithms for non linear optimization, online.

– J. F. Bonnans, Optimisation continue : cours et exercices, Dunod, 2006.
– H. B. Hiriart-Urruty and C. Lemaréchal, Convex analysis and minimization algorithms, Vol. I, II, Springer-Verlag, 1993.

Analysis of partial differential equations

Institution: Department of Mathematics, UVSQ
Teaching hours: Lectures: 24h Tutorials: 24h
ECTS: 6
Semester: 1
Nature: Compulsory
Language: French
Faculty members: Emmanuelle Crépeau
Location: UVSQ
Specializations: Modelling and simulation
Examination: midterms and final exam

Prerequisites: Integral calculus, differential calculus, topology

Description

This course introduces some tools for the analysis of Partial Differential Equations. We will give first some basic results on functional analysis and on Sobolev spaces. Then we will study some fundamental equations like the wave equation and the Schrödinger one.

Syllabus

- Basic results on functional analysis
  - further results on Banch spaces
  - linear forms, duality.
  - Linear continuous applications
  - Hahn-Banach Theorem
  - open mapping Theorem, and closed graph Theorem .
  - Weak and weak $\star$ convergences.
- L^p Spaces and Sobolev spaces.
- Variationnal formulation.
- Wave equation.
- Schrödinger equation.

References

1 Rudin, W. (1973). Functional analysis, 1973.
2 Lawrence C. Evans, Partial differential equations, Graduate Studies in Mathematics, Vol. 19, AMS.

Advanced numerical methods and scientific computing

Institution: Department of Mathematics, UVSQ
Teaching hours: Lectures: 24h Tutorials: 24h
ECTS: 6
Semester: 2
Nature: obligatoire
Faculty members: Christophe Chalons
Location: UVSQ
Language: French
Nature: Compulsory
Specializations: Modelling and simulation, Mathematical Methods for Mechanics
Examination: midterms and final exam

Prerequisites: Applied functional analysis, distributions and Sobolev spaces (suggested but not mandatory)

Description

The objective of this course is to provide an introduction to the mathematical analysis and the numerical approximation of the solutions of some partial differential equations (PDE). These equations are naturally present in many applications like mechanical and physical engineering (nuclear, aerospace, petroleum engineering, automotive...) or finance, economics, chemistry...We will present some of the most important results of theoretical analysis but also the three major classes of numerical methods (finite element method, finite volume method and finite volume method). Another objective of the course is to provide the students with a practical experience in the numerical solving of PDEs by means of computer programming sessions.

Syllabus

- Elliptic equations
  - Reminders on distributions and Sobolev spaces
  - Variational formulation
  - Lax-Milgram theorem
  - Introduction to the finite element method in 1D and 2D
- Hyperbolic equations
  - Transport equation and wave equation
  - Introduction to the finite volume method
- Parabolic equations
  - Heat equation
  - Introduction to the finite difference method

References

1 Pierre-Arnaud Raviart, Jean-Marie Thomas : Introduction à l'analyse numérique des équations aux dérivées partielles, éditions Dunod 1998.

2 Brigitte Lucquin : Equations aux dérivées partielles et leurs approximations, Ellipses, 2004.

3 E. Godlewski et P.-A. Raviart : Hyperbolic systems of conservation laws, Ellipses 1991.

4 Lawrence C. Evans : Partial differential equations, Graduate Studies in Mathematics, Vol. 19, AMS.

5 C. Strikwerda : Finite Difference Schemes and Partial Differential Equations, SIAM 2004.

6 L. Hörmander : Lectures on Nonlinear Hyperbolic Differential Equations, Springer 1997.

7 F. Lagoutière : Polycopié de cours sur les Equations aux dérivées partielles et leurs approximations, Université Paris-Sud.

Stability and bifurcation

Institution: Department of Mathematics, UVSQ
Teaching hours: Lectures: 12h Tutorials: 12h
ECTS: 3
Semester: 2
Faculty members: Paolo Vannucci
Location: UVSQ
Language: French or english
Nature: Compulsory
Specializations: Modelling and simulation, Mathematical Methods for Mechanics
Examination: midterms and final exam
Prerequisites: basic elements of calculus, calculus of variations, dynamics.

Description

Many problems in physics, mechanics, biology, industry needs the solution of ODEs or PDEs. This course gives some modern mathematical techniques for the qualitative study of these solutions. These properties concern the dependance on the initial conditions, the stability of equilibrium points and sensitivity with respect to parameters. The course will refer to several problems of mechanics, physics, biology.

Syllabus

- Introduction
  Some examples of stability and bifurcation : stability of the orbits in a field with potential $1/r^2$; rigid beams with elastic pin-joints ; different approaches. Stability in the sense of Hadamard. Short history. Recall of some basic notions : Lagrange equations, equilibrium conditions.
- Stability :
  The space of phases : trajectories, classification of equilibrium points. Notion of stability, attractors. Duffing equation, Van der Pol oscillator.
- Bifurcation :
  Basic notions. Branching diagram. Limit points and bifurcation points. Geometric and algebraic points of view. Snap-through, types of bifurcation. Bifurcation modes.
- Hopf bifurcation :
  Definition. Hopf bifurcation and stability. Lorentz equation. Types of branching.
- Liapounov stability :
  Definition. Liapounov stability theorem. The direct method of Liapounov. Asymptotic stability.
- Stability of the equilibrium of conservative systems :

Equilibrium as potential stationary point. Lagrange-Dirichlet theorem. Poincaré's coefficients of stability. Perturbation effects.
– Bifurcation and stability of the equilibrium of elastic beams :
The Euler beam. Prandtl lateral instability. Torsion-bending instability.
– Energetic approaches :
General principles. Rayleigh quotient. Ritz method. Galerkin method.

References

– S. Timoshenko, S. Gere : *Theory of elastic stability.* McGraw-Hill, 1961.
– N. Chetaev : *The stability of motion.* Pergamon Press, 1961.
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Modelling in Mechanics

Institution: Departement of mathematics, UVSQ
Teaching hours: Lectures: 27h Tutorials: 42h
ECTS: 6
Semester: S2
Location: Paris-Sud University, Orsay
Nature: Compulsory
Language: French
Faculty members: Frédéric Lagoutière
Location: UVSQ
Specializations: Modelling and simulation, Mathematical Methods for Mechanics

Prerequisites: basics of ordinary differential equations and numerica analysis. Variational formulations of PDEs. Basic concepts of programming.

Description

This course aims to use mathematical tools in practical situations in mechanics. In the first part focus is on mass-spring systems which allow to gain more insight about basic notions of mechanics like forces, energy, work and power. This approach leads to the modeling of various phenomena. The arising models could then be studiend by means of the theory of differential equations.

The second part is devoted to studying fluid models. First, focus will be on perfect fluids (the model of a perfect fluid gives a good approximation of the flow around an aircraft wing or around the keel of a boat). Then, we will deal with real fluids which display some resistance to deformations. Depending on the amount of time available, more complex situations could be considered (for example flow in porous media, biological flows, etc.).

The course combines conceptual presentations with practical hands-on computer sessions.

Professors

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Courses in master 1 :
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Courses in master 1 :
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